Fault Tolerant Quantum Error Correction

Master's Thesis Midterm Presentation

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Overview

- Introduction to Quantum Error Correction
- Fault Tolerance and Detector Error Models
- Research Gap
- Simulation Methodology

Quantum Computing

- Simulating quantum systems on classical hardware is exponentially complex
  $\rightarrow$ Can't we use quantum hardware to simulate quantum systems? [Fey82]
- Some problems that are "hard" to solve on classical computers we can "easily" solve on quantum computers [Pre18]

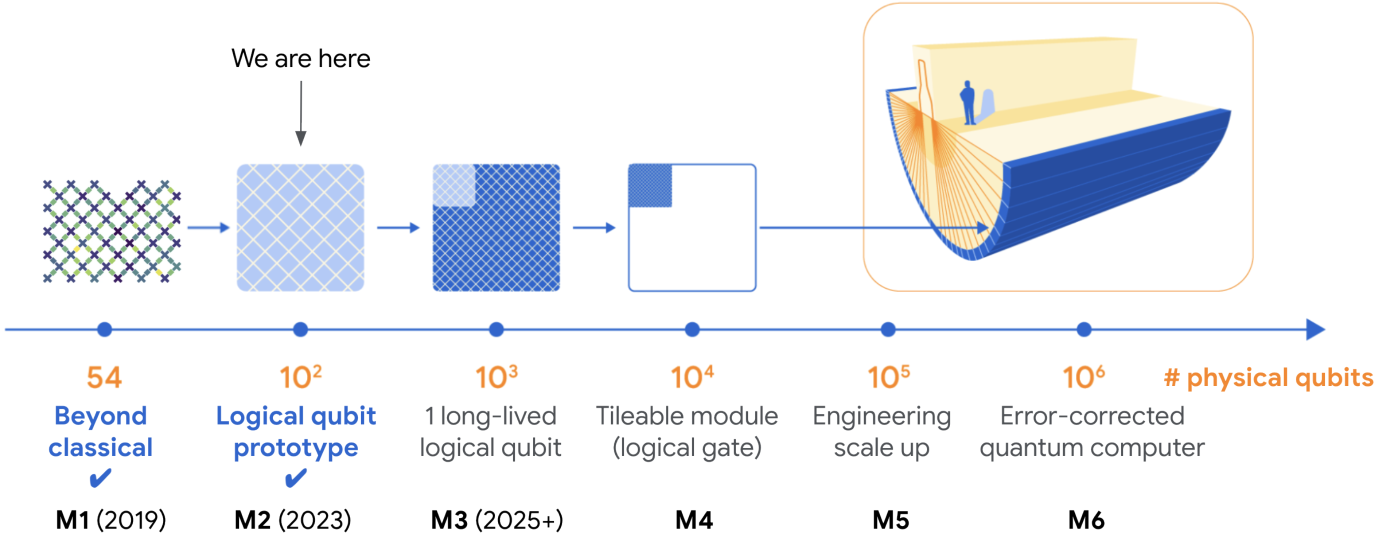


Figure: Google Quantum AI's quantum computing roadmap [Goo].

[Fey82] R. P. Feynman, "Simulating physics with computers," en, *International Journal of Theoretical Physics*, vol. 21, no. 6, pp. 467–488, Jun. 1982,
[Pre18] J. Preskill, "Quantum Computing in the NISQ era and beyond," en-GB, *Quantum*, vol. 2, p. 79, Aug. 2018,
[Goo] Google Quantum AI, *Quantum Computing Roadmap*, en. Accessed: Jan. 28, 2026.

The Need for Quantum Error Correction

- Errors during quantum computation are inevitable because quantum systems are fragile
- We want to interact with the quantum state but not disturb it
- We employ more physical qubits to introduce redundancy and use the resulting *physical* state to represent the *logical* state [Rof19]

- Typical scales
  - IBM recently introduced a scheme encoding 12 logical qubits in 288 physical ones [BCG+24]
  - The physical error rate is typically assumed to be 10^{-3} for simulations (e.g., [BCG+24])
  - Decoding has to happen with ultra-low latency to avoid the backlog problem (about 1 us per data extraction round) [CSB+24]

[Rof19] J. Roffe, "Quantum error correction: An introductory guide," *Contemporary Physics*, vol. 60, no. 3, pp. 226–245, Jul. 2019,

[BCG+24] S. Bravyi et al., "High-threshold and low-overhead fault-tolerant quantum memory," *Nature*, vol. 627, no. 8005, pp. 778–782, Mar. 2024,

[CSB+24] L. Caune et al., *Demonstrating real-time and low-latency quantum error correction with superconducting qubits*, Oct. 2024. Accessed: Jan. 28, 2026.

Peculiarities of the Quantum Setting

- Quantum error correction (QEC) is actually able to protect the actual quantum state
- Similar to bits and gates, quantum systems are built on top of qubits and quantum gates
- We have to consider phase flip errors in addition to bit flip errors [Rof19]

$$\begin{aligned} |0\rangle &\to |1\rangle \\ |1\rangle &\to |0\rangle \end{aligned}$$

(a) Bit flip (X) error

$$\begin{aligned} |0\rangle &\to |0\rangle \\ |1\rangle &\to -|1\rangle \end{aligned}$$

(b) Phase flip (Z) error

$$\begin{aligned} |0\rangle &\to |1\rangle \\ |1\rangle &\to -j\,|0\rangle \end{aligned}$$

(c) Y error: Combination of X and Z

- Measuring the qubits directly destroys superpositions and entanglement
  $\to$ We generally only work with the syndrome, which we can measure [NC10]
- Sometimes superposition permits multiple equivalent solutions to the decoding problem (*quantum degeneracy*) [RWBC20]

[NC10] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition*, en. Cambridge: Cambridge University Press, Dec. 2010, ISBN: 978-0-511-97666-7.

[Rof19] J. Roffe, "Quantum error correction: An introductory guide," *Contemporary Physics*, vol. 60, no. 3, pp. 226–245, Jul. 2019,

[RWBC20] J. Roffe, D. R. White, S. Burton, and E. Campbell, "Decoding across the quantum low-density parity-check code landscape," *Physical Review Research*, vol. 2, no. 4, p. 043 423, Dec. 2020.

Stabilizer and Calderbank Shor Steane Codes

- Stabilizer codes [NC10]
  - The code space can implicitly be defined using *stabilizer generators*
  - We can represent them using parity check matrices
  - Quantum analog of linear codes

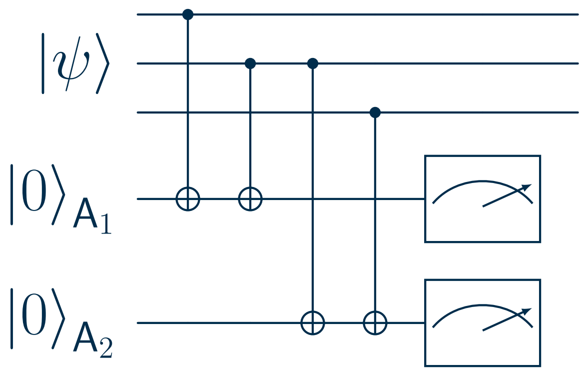
- Calderbank Shor Steane (CSS) codes [NC10]
  - Subset of stabilizer codes
  - Can correct X and Z errors independently
  - Described using two separate parity check matrices $\boldsymbol{H}_{\mathrm{X}}$ and $\boldsymbol{H}_{\mathrm{Z}}$
  - Can be constructed from two binary linear codes $\mathcal{C}_1\,[n, k_1]$ and $\mathcal{C}_2\,[n, k_2]$ with $\mathcal{C}_2 \subset \mathcal{C}_1$

[NC10] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition*, en. Cambridge: Cambridge University Press, Dec. 2010, ISBN: 978-0-511-97666-7.

Syndrome Extraction Circuits

- We entangle the state with *ancilla qubits* to perform syndrome measurements [NC10]
- Example: The 3-qubit repetition code[1]

$$\boldsymbol{H} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$



[NC10] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition*, en. Cambridge: Cambridge University Press, Dec. 2010, ISBN: 978-0-511-97666-7.

[1] Note that, for simplicity, this chosen example is a code that is only able to correct X errors (bit flips)

Overview

- **Introduction to Quantum Error Correction**
- **Fault Tolerance and Detector Error Models**
    - Fault Tolerance
    - Detector Error Models
- **Research Gap**
- **Simulation Methodology**

Fault Tolerance

- The quantum gates we use for syndrome extraction are faulty themselves
  $\rightarrow$ We need *fault-tolerant* QEC
- A QEC procedure is said to be fault tolerant if, in addition to correcting *input errors*, the spread of *internal errors* is sufficiently limited [DTTBE25]

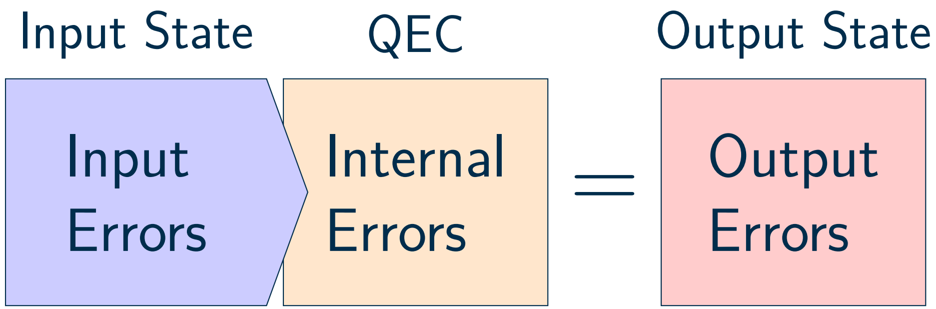


Figure: Overview of the flow of errors in a QEC system. Adapted from [DTTBE25].

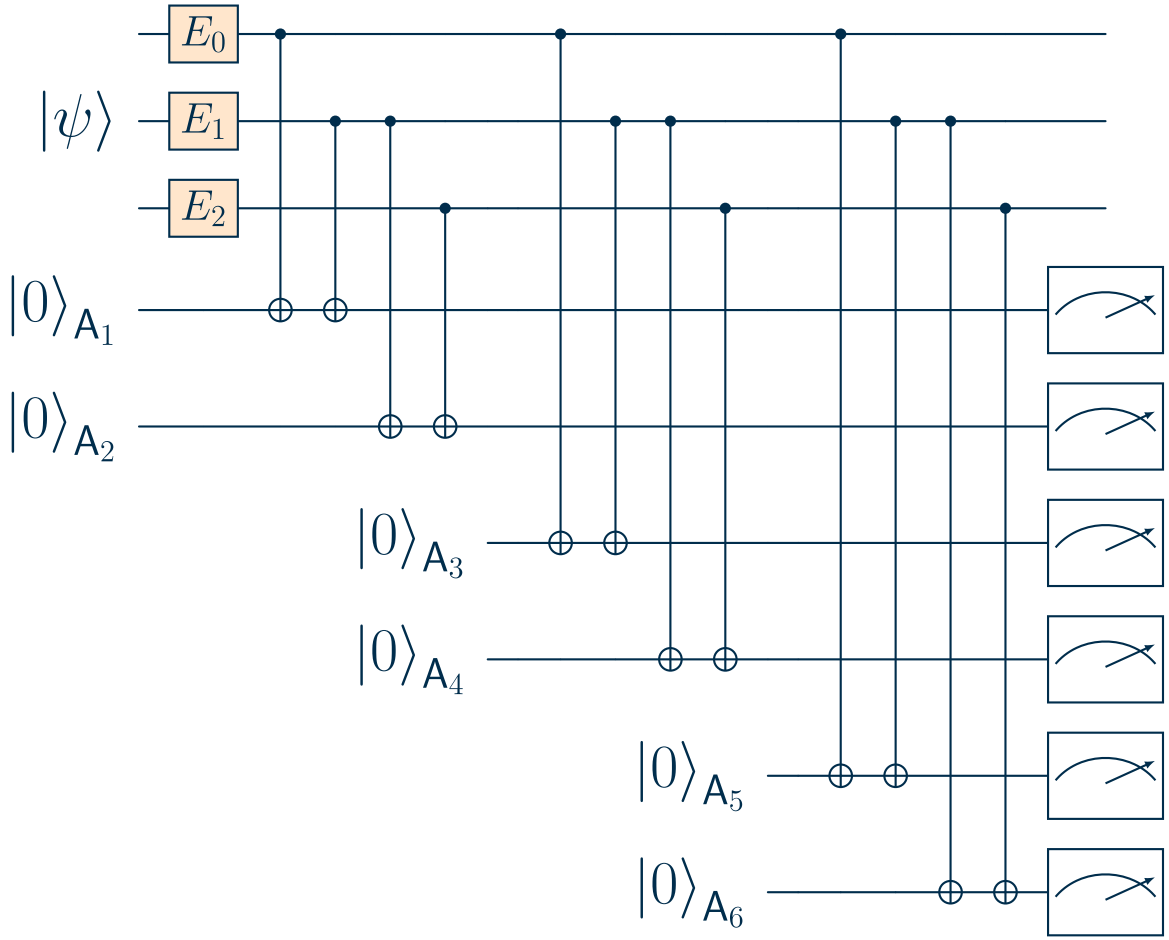
- We have to modify the syndrome extraction circuitry to be fault tolerant (e.g., by using specially prepared multi-qubit states for each ancilla [Sho97])
- We generally perform multiple rounds of syndrome extraction

[Sho97] P. W. Shor, “Fault-tolerant quantum computation,”, Mar. 1997,
[DTTBE25] P.-J. H. S. Derks, A. Townsend-Teague, A. G. Burchards, and J. Eisert, “Designing fault-tolerant circuits using detector error models,”, Oct. 2025,

The Measurement Syndrome Matrix I

- Each column of the *measurement syndrome matrix* Ω corresponds to a measurement pattern an error produces [DTTBE25]
- Example: 3-qubit repetition code (Only bit flips on data qubits)

$$\boldsymbol{\Omega} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$



[DTTBE25] P.-J. H. S. Derks, A. Townsend-Teague, A. G. Burchards, and J. Eisert, "Designing fault-tolerant circuits using detector error models,", Oct. 2025,

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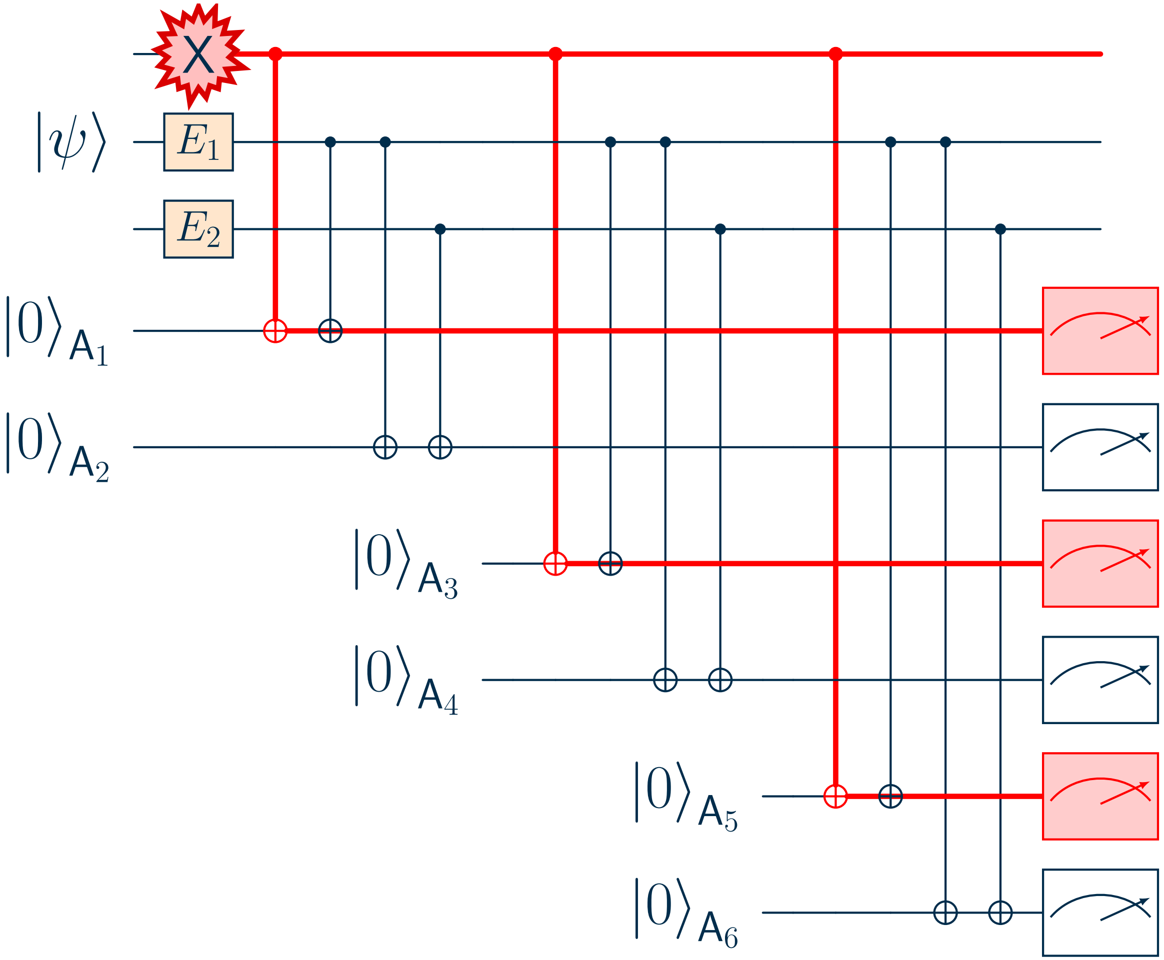

[DTTBE25] P.-J. H. S. Derks, A. Townsend-Teague, A. G. Burchards, and J. Eisert, "Designing fault-tolerant circuits using detector error models,", Oct. 2025,

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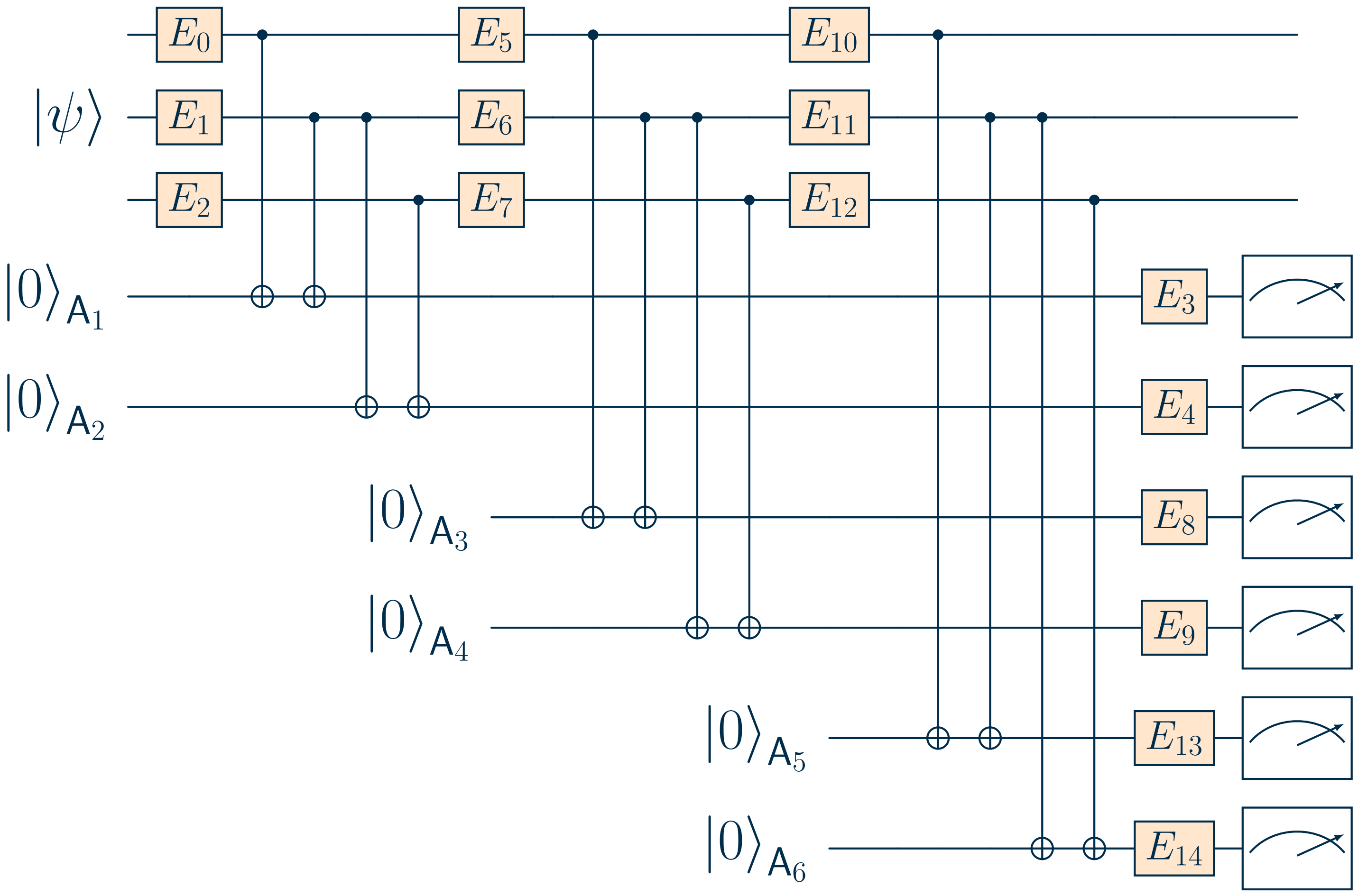
- Each column of the *measurement syndrome matrix* Ω corresponds to a measurement pattern an error produces [DTTBE25]
- Example: 3-qubit repetition code (Phenomenological noise [DTTBE25])

$$\Omega = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$



[DTTBE25] P.-J. H. S. Derks, A. Townsend-Teague, A. G. Burchards, and J. Eisert, "Designing fault-tolerant circuits using detector error models,", Oct. 2025,

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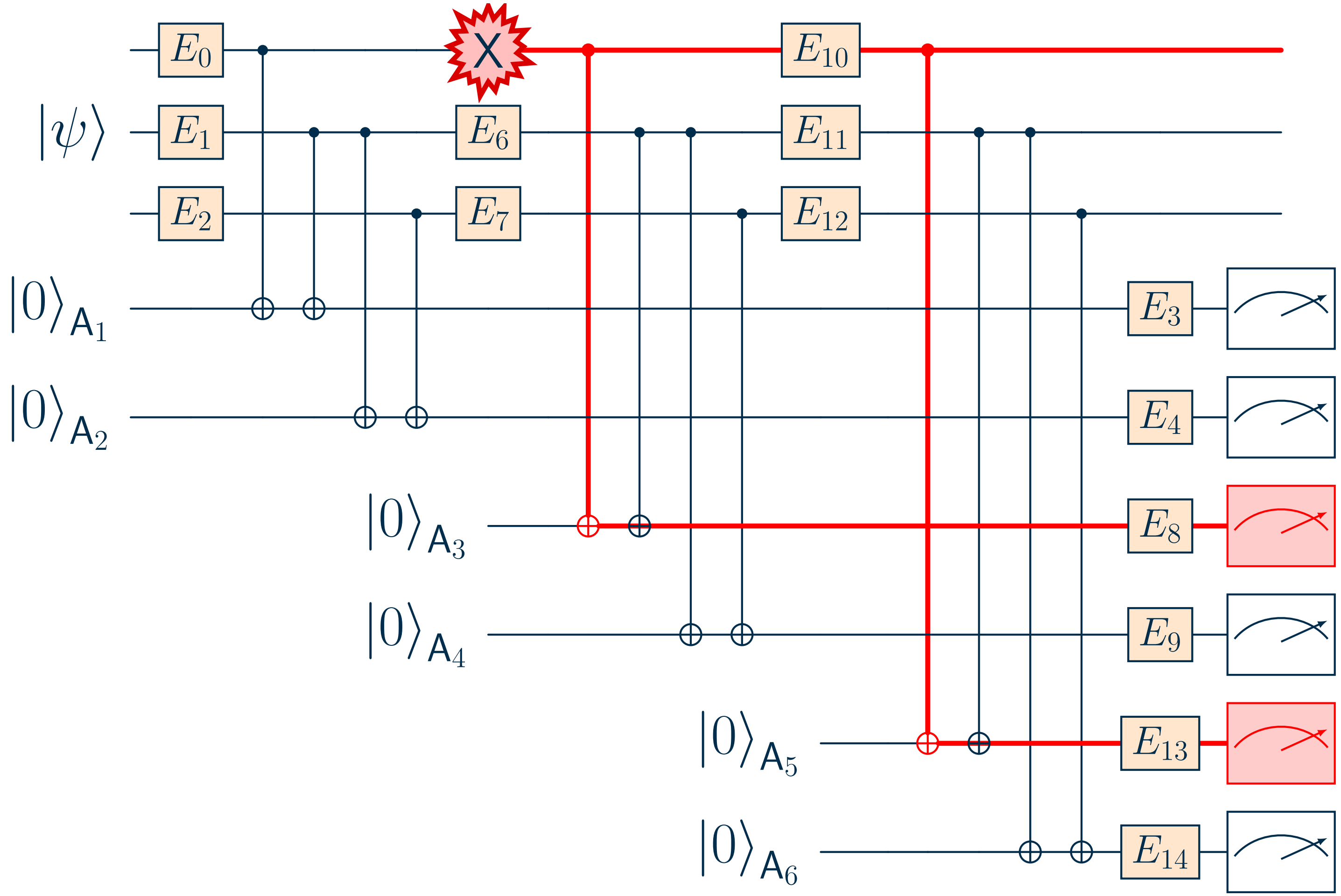

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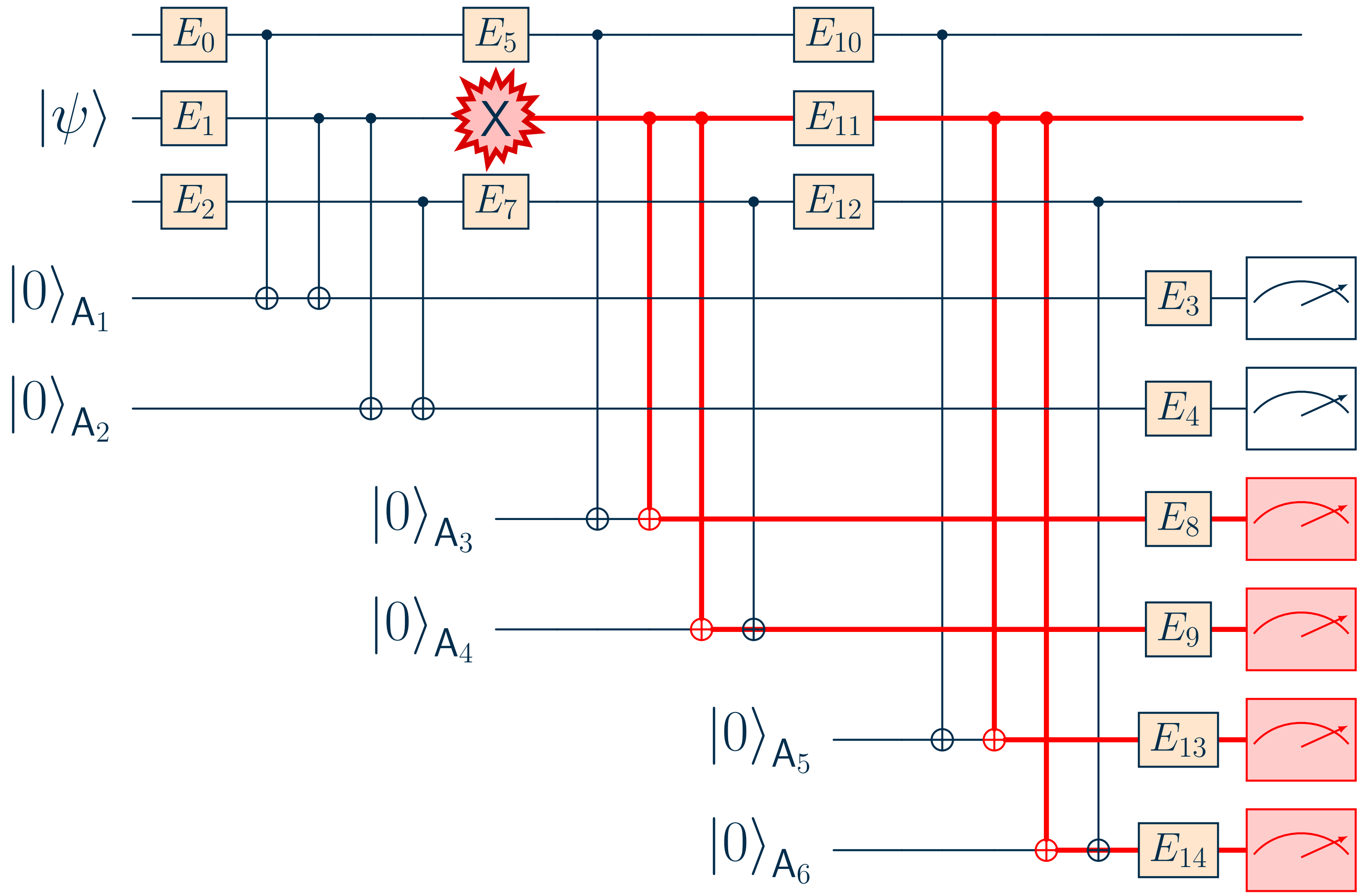
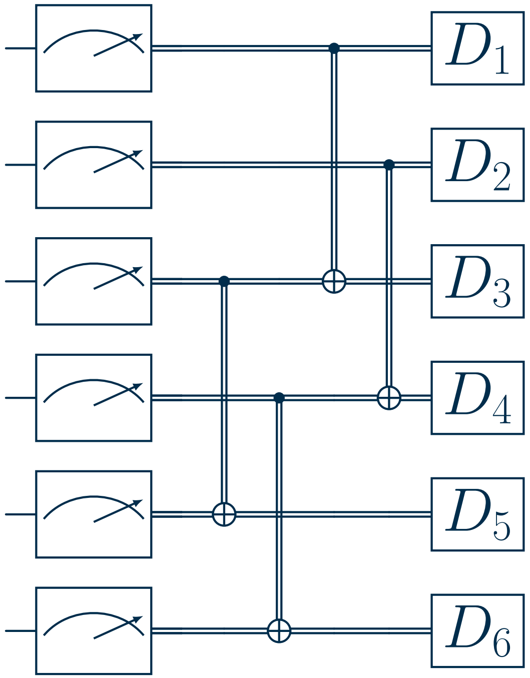


[DTTBE25] P.-J. H. S. Derks, A. Townsend-Teague, A. G. Burchards, and J. Eisert, "Designing fault-tolerant circuits using detector error models,", Oct. 2025,

The Detector Error Matrix I

- A detector is a parity constraint on a set of measurement outcomes [DTTBE25]
- Each column of the *detector error matrix* $\boldsymbol{H}$ corresponds to a detector pattern an error produces
- We can mitigate the propagation of errors into subsequent rounds by XORing the measurements, i.e., defining detectors appropriately; this is equivalent to row additions

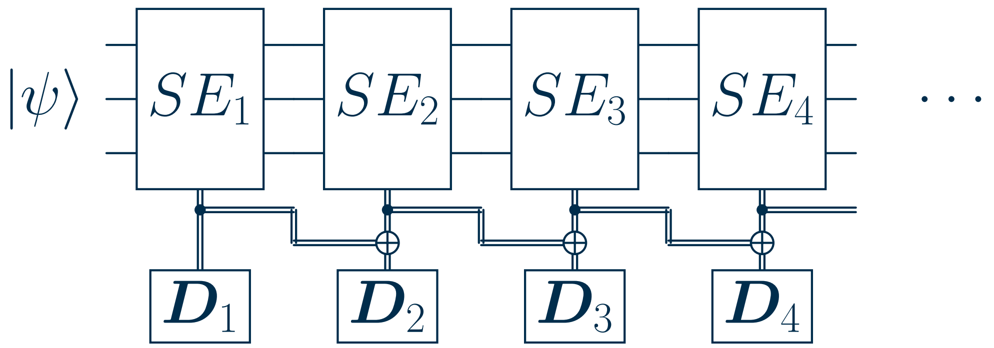


$$\boldsymbol{H} = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

[DTTBE25] P.-J. H. S. Derks, A. Townsend-Teague, A. G. Burchards, and J. Eisert, “Designing fault-tolerant circuits using detector error models,”, Oct. 2025,

The Detector Error Matrix II

- Visualization of general process



- E.g., for bivariate bicycle (BB) codes, the resulting detector error matrix under circuit-level noise has the form [GCR24]

$$\boldsymbol{H} = \begin{pmatrix} \boldsymbol{H}_0 & \boldsymbol{H}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots \\ \boldsymbol{0} & \boldsymbol{H}_2 & \boldsymbol{H}_0 & \boldsymbol{H}_1 & \boldsymbol{0} & \boldsymbol{0} & \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{H}_2 & \boldsymbol{H}_0 & \boldsymbol{H}_1 & \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{H}_2 & \\ \vdots & & & & & & \ddots \end{pmatrix}$$

[GCR24] A. Gong, S. Cammerer, and J. M. Renes, *Toward Low-latency Iterative Decoding of QLDPC Codes Under Circuit-Level Noise*, en, Mar. 2024. Accessed: Nov. 24, 2025.

Noise Model Types

- The noise model assigns a likelihood to the occurrence of each error
- The *depolarizing channel* considers [NC10]
  - X, Y or Z errors on the data qubits
- *Phenomenological noise* considers [DTTBE25]
  - X errors on data qubits before each measurement round
  - X errors on measurement outcomes
- *Circuit-level noise* considers [DTTBE25]
  - X, Y or Z errors after state preparation
  - n-qubit X, Y or Z errors after any n-qubit gate
  - X errors on measurement outcomes

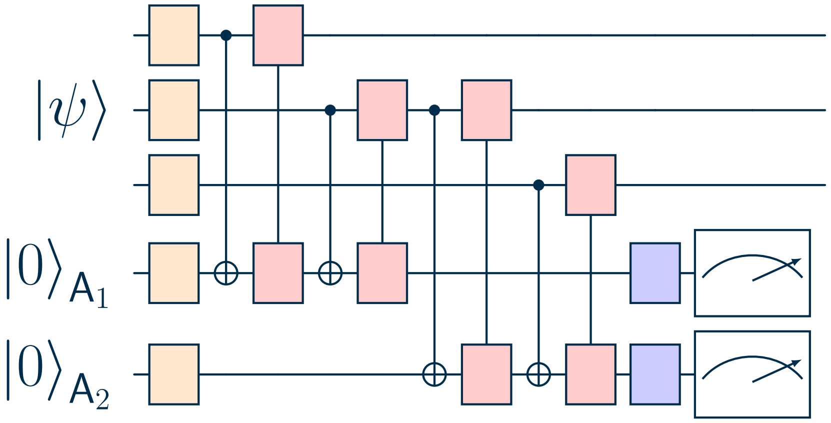


Figure: Circuit-level noise model for the 3-qubit repetition code (for X errors).

[NC10] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition*, en. Cambridge: Cambridge University Press, Dec. 2010, ISBN: 978-0-511-97666-7.

[DTTBE25] P.-J. H. S. Derks, A. Townsend-Teague, A. G. Burchards, and J. Eisert, "Designing fault-tolerant circuits using detector error models,", Oct. 2025,

Decoding using Detector Error Models

- A detector error model (DEM) combines a detector error matrix and a noise model
- When employing belief propagation (BP), the syndrome-based variant must be used **[BBA+15]**
- The likelihoods of different error locations can be used as priors for decoding

- Challenges
  - Repeated syndrome measurements come with increased decoding complexity **[GCR24]**
  - Degeneracy and short cycles lead to degraded performance of BP **[BBA+15]**

[BBA+15] Z. Babar et al., "Fifteen Years of Quantum LDPC Coding and Improved Decoding Strategies," *IEEE Access*, vol. 3, pp. 2492–2519, 2015,

[GCR24] A. Gong, S. Cammerer, and J. M. Renes, *Toward Low-latency Iterative Decoding of QLDPC Codes Under Circuit-Level Noise*, en, Mar. 2024. Accessed: Nov. 24, 2025.

Overview

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- **Research Gap**
  - State of the Art
  - Proposed Research
- **Simulation Methodology**

Addressing the Challenges

- Many window-based approaches exist to combat the decoding complexity
  - Parallel decoding of syndromes [SBB+23]
  - Using sliding windows [HP23] [GCR24]
- To deal with the degraded BP performance, it is usually modified or extended
  - Ordered statistics decoding (OSD) post-processing [RWBC20]
  - Guided decimation [GCR24]
  - Neural approaches [KL22] [MSLS25]
  - Ensemble decoding [KSWB25]

[RWBC20] J. Roffe, D. R. White, S. Burton, and E. Campbell, "Decoding across the quantum low-density parity-check code landscape," *Physical Review Research*, vol. 2, no. 4, p. 043 423, Dec. 2020.

[KL22] K.-Y. Kuo and C.-Y. Lai, "Exploiting degeneracy in belief propagation decoding of quantum codes," en, *npj Quantum Information*, vol. 8, no. 1, p. 111, Sep. 2022,

[HP23] S. Huang and S. Puri, "Improved Noisy Syndrome Decoding of Quantum LDPC Codes with Sliding Window,", Nov. 2023,

[SBB+23] L. Skoric et al., "Parallel window decoding enables scalable fault tolerant quantum computation," en, *Nature Communications*, vol. 14, no. 1, p. 7040, Nov. 2023,

[GCR24] A. Gong, S. Cammerer, and J. M. Renes, *Toward Low-latency Iterative Decoding of QLDPC Codes Under Circuit-Level Noise*, en, Mar. 2024. Accessed: Nov. 24, 2025.

[MSLS25] S. Miao, A. Schnerring, H. Li, and L. Schmalen, "Quaternary Neural Belief Propagation Decoding of Quantum LDPC Codes with Overcomplete Check Matrices," *IEEE Access*, vol. 13, pp. 25 637–25 649, Feb. 2025,

[KSWB25] S. Koutsioumpas, H. Sayginel, M. Webster, and D. E. Browne, "Automorphism Ensemble Decoding of Quantum LDPC Codes," en, Mar. 2025

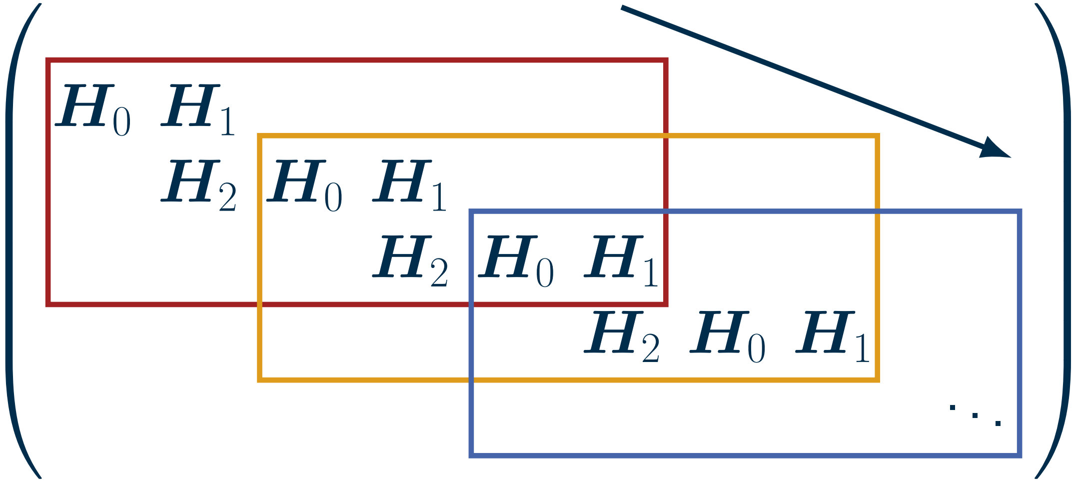
Sliding-Window Decoding

- The sliding window approach taken in [GCR24] resembles a spatially coupled low density parity check (SC-LDPC) code decoder
- However, they don't pass soft information between windows (only update of syndrome based on hard decision in previous window)

$$\begin{pmatrix} \boldsymbol{H}_0 & \boldsymbol{H}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{H}_2 & \boldsymbol{H}_0 & \boldsymbol{H}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{H}_2 & \boldsymbol{H}_0 & \boldsymbol{H}_1 \end{pmatrix} \begin{pmatrix} \hat{\boldsymbol{e}}_0 \\ \vdots \\ \hat{\boldsymbol{e}}_5 \end{pmatrix} = \begin{pmatrix} \boldsymbol{s}_1 \\ \boldsymbol{s}_2 \\ \boldsymbol{s}_3 \end{pmatrix}$$

$$\boldsymbol{s}_2' = \boldsymbol{s}_2 + \boldsymbol{H}_2 \hat{\boldsymbol{e}}_1$$

(a) Equations for the decoding of the first window



(b) Visualization of sliding window procedure

- They try BP + OSD and a modification of BP with guided decimation

[GCR24] A. Gong, S. Cammerer, and J. M. Renes, *Toward Low-latency Iterative Decoding of QLDPC Codes Under Circuit-Level Noise*, en, Mar. 2024. Accessed: Nov. 24, 2025.

Proposed Research

- Research gap
  - Existing literature into circuit-level noise fails to properly consider the SC-LDPC like structure of the detector error matrix
- Proposed Methodology
  - Adapt modified guided decimation decoder from **[GCR24]** to pass soft information
  - Investigate performance of different modifications of BP for "inner decoder" (e.g., quaternary neural BP **[MSLS25]**)

[GCR24] A. Gong, S. Cammerer, and J. M. Renes, *Toward Low-latency Iterative Decoding of QLDPC Codes Under Circuit-Level Noise*, en, Mar. 2024. Accessed: Nov. 24, 2025.

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  - The Code and Other Parameters
  - Proposed Methodology

Memory and Stability experiments

- What is a memory experiment? Communications Engineering view (what are my inputs and outpus? What do I expect?)
- What is a stability experiment?

Noise Models and Figures of Merit

- For circuit-level noise, often, all error probabilities are set to the same value for simulations [FSG09]
- There are other approaches (e.g., SDMB noise, SI noise) [DTTBE25]

- Footprint plots
- Other figure of merit (Look into ECCentric?)

[DTTBE25] P.-J. H. S. Derks, A. Townsend-Teague, A. G. Burchards, and J. Eisert, "Designing fault-tolerant circuits using detector error models,", Oct. 2025,

[FSG09] A. G. Fowler, A. M. Stephens, and P. Groszkowski, "High-threshold universal quantum computation on the surface code," *Physical Review A*, vol. 80, no. 5, p. 052 312, Nov. 2009,

Proposed Simulation Methodology

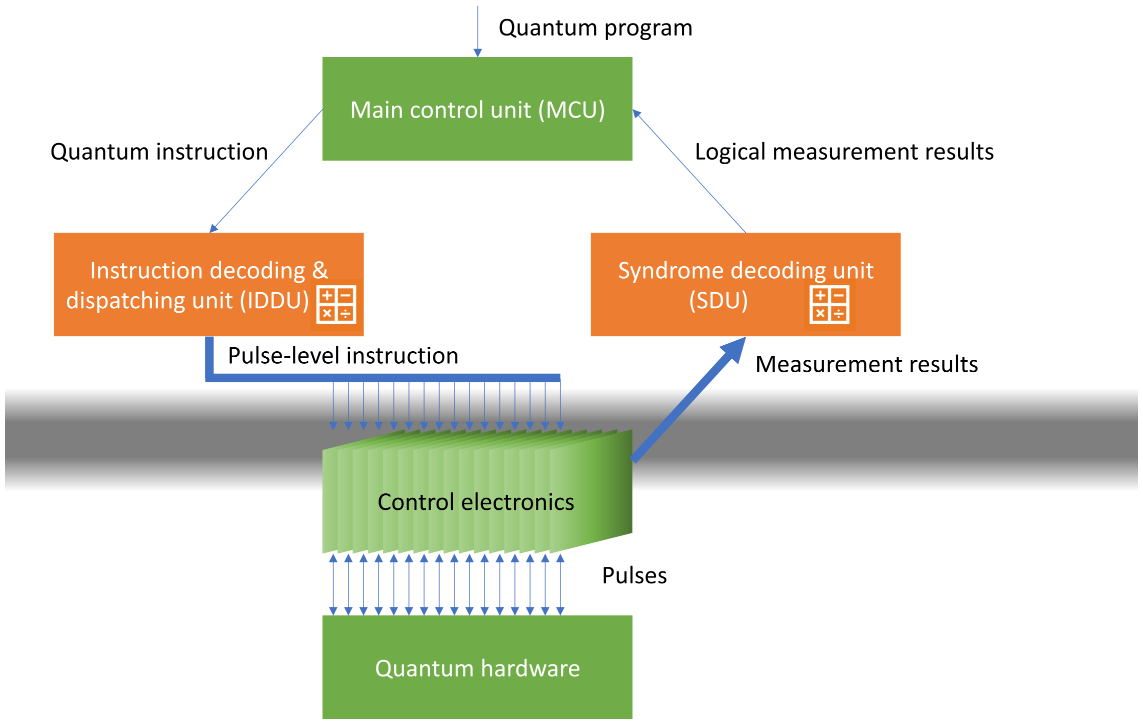
- Noise model
- Memory or stability experiment
- Figure of merit: Footprint plot
- Comparison with BB code also simulated by [GCR24]
- Comparison with surface code

[GCR24] A. Gong, S. Cammerer, and J. M. Renes, *Toward Low-latency Iterative Decoding of QLDPC Codes Under Circuit-Level Noise*, en, Mar. 2024. Accessed: Nov. 24, 2025.

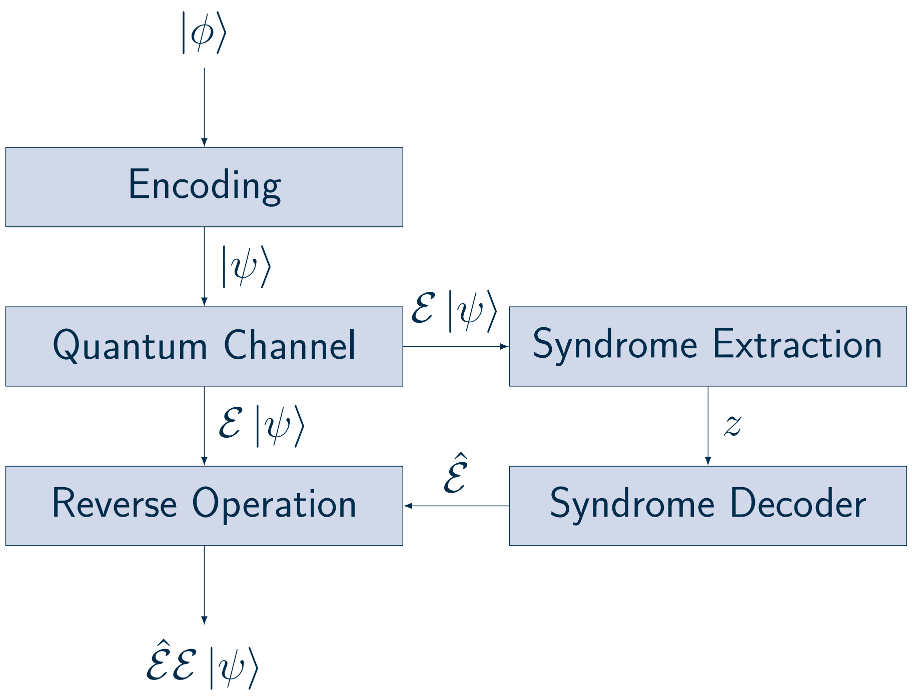
Questions

Thank you for your attention!
Any questions?

System Level Overview



(a) Schematic workflow of surface code quantum computation **[ZZC+23]**.



(b) Block diagram of QEC using stabilizer codes **[MSLS25]**.

[ZZC+23] F. Zhang et al., "A Classical Architecture for Digital Quantum Computers," *ACM Transactions on Quantum Computing*, vol. 5, no. 1, 3:1–3:24, Dec. 2023,

[MSLS25] S. Miao, A. Schnerring, H. Li, and L. Schmalen, "Quaternary Neural Belief Propagation Decoding of Quantum LDPC Codes with Overcomplete Check Matrices," *IEEE Access*, vol. 13, pp. 25 637–25 649, Feb. 2025,