HiWi Notes: Minimization of the Code Constraint Polynomial using Homotopy Continuation Methods

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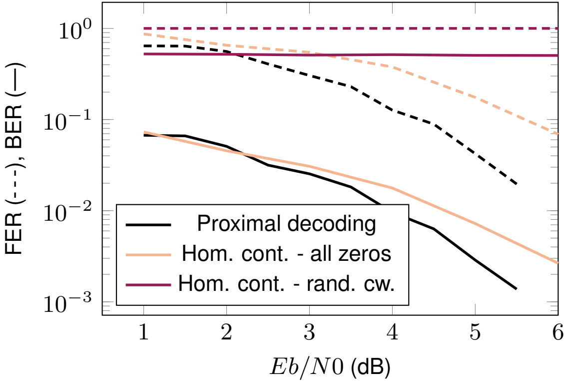
The All-Zeros Assumption

- Previous results were generated using the all-zeros assumption
- To cancel out any numerical effects, subsequent results have been generated using randomly generated codewords
- Using randomly generated codewords paints a very different picture



(a) BCH(31,26) Code

	Parameter	Value
n_{iter}	for homotopy continuation	20
n_{iter}	for Newton corrector	5
δ_{max}	for Newton corrector	0.01
Δs	for Euler predictor	0.05
n_{retries}	for Euler predictor	5

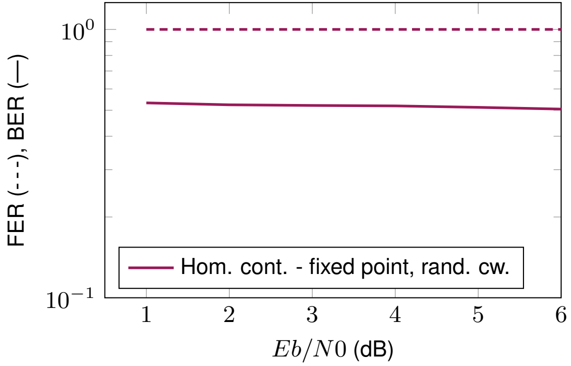
Fixed Point Homotopy

- Previous results were generated using the Newton homotopy

$$G(\boldsymbol{x}) = F(x) - F(\boldsymbol{y}) \quad \Rightarrow \quad H(\boldsymbol{x}) = F(\boldsymbol{x}) - (1-t)F(\boldsymbol{y})$$

- We could instead try the fixed point homotopy

$$G(\boldsymbol{x}) = (\boldsymbol{x} - \boldsymbol{y}) \quad \Rightarrow \quad H(\boldsymbol{x}) = (1-t)(\boldsymbol{x} - \boldsymbol{y}) + tF(\boldsymbol{x})$$



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Parameter Exploration

- Fixed parameters:

Parameter		Value
E_b/N_0		$6\,\mathrm{dB}$
n_{iter}	for homotopy continuation	200
n_{iter}	for Newton corrector	20
n_{retries}	for Euler predictor	20

- Random codewords, Newton homotopy, BCH(31, 26) code
- Low decoding failure rate (DFR) $\rightarrow$ We do often reach codewords, just not the correct ones

DFR		Euler step size						
		0.001	0.005	0.01	0.05	0.1	0.5	1.0
Newtown threshold	0.001	0.070	0.055	0.085	0.070	0.075	0.050	0.060
	0.005	0.055	0.075	0.060	0.070	0.100	0.070	0.050
	0.01	0.065	0.060	0.075	0.085	0.095	0.040	0.045
	0.05	0.090	0.080	0.085	0.085	0.045	0.065	0.055
	0.1	0.100	0.085	0.045	0.080	0.065	0.055	0.075
	0.5	0.070	0.075	0.125	0.055	0.070	0.055	0.090
	1.0	0.055	0.095	0.060	0.050	0.055	0.075	0.055

FER		Euler step size						
		0.001	0.005	0.01	0.05	0.1	0.5	1.0
Newtown threshold[1]	0.001	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.005	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.01	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.05	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

BER		Euler step size						
		0.001	0.005	0.01	0.05	0.1	0.5	1.0
Newtown threshold	0.001	0.497	0.496	0.509	0.495	0.508	0.495	0.503
	0.005	0.509	0.503	0.491	0.502	0.506	0.502	0.502
	0.01	0.498	0.492	0.520	0.509	0.500	0.502	0.504
	0.05	0.503	0.499	0.507	0.494	0.492	0.500	0.506
	0.1	0.499	0.499	0.500	0.508	0.502	0.488	0.507
	0.5	0.510	0.503	0.510	0.495	0.507	0.515	0.501
	1.0	0.502	0.506	0.506	0.502	0.500	0.505	0.500

[1] "Newton threshold" refers to the threshold used for the convergence criterion of the newton corrector

Tentative Conclusion

- Original motivation
  - Novel decoding technique
  - Possible decoding guarantees due to homotopy continuation behavior
- Simulation results
  - Implementing a working decoder has proven difficult, irrespective of the parameters / code chosen
  - The homotopy continuation implementation itself seems to be working, judging by the DFR
  - There are two alternatives that both describe the observed results:
    - We always converge to the same codeword - error rates are due to Tx codewords being random
    - We randomly converge to codewords that have nothing to do with what was sent
- Problems
  - It is not obvious how we can apply the theory from homotopy continuation to the decoder
  - Decoding algorithm seems too comput. expensive (matrix inversion necessary for each iteration)
- Conclusion
  - There is no hard evidence yet that a decoder based on homotopy continuation can't work
  - However, pursuing this line of research doesn't seem particularly promising, due to missing motivation and missing evidence it can work at all